

## The Computational Model of Financial Portfolios in the CAPM

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**Abstract:** The Republic of Kazakhstan is integrated in the global system as a country with a small open economy. The main exogenous financial performance: the structure and profitability of portfolios of risky assets formed in the external environment, in the global financial markets does not depend on the behavior of countries. Optimizing of financial portfolios of risky assets is the basis of modern theories of financial portfolios in the market of fixed capital. The goal of building an investment portfolio is particularly relevant for investment institutions, i.e. organizations professionally involved in accumulation of financial resources of their clients and use them in the capital market. Effective management of financial assets of the company involves the development and use of various models of investment portfolio. Such a model is developed in the mid 60s by William Sharpe and John Lintern and was named the valuation of financial assets (Capital Asset Pricing Model-CAPM). The article deals with one aspect of the theory of investment portfolio - the formation of efficient portfolio with restrictions on the share of its constituent assets. The paper presents the basic models and algorithms for solving problems in the interpretation of the pension fund.

**Key words:** The pension system • Return on assets • Securities • Risk • Investment • Demand price • The offer price • The volume of demand • The volume of supply • Financial portfolio • CAPM

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### INTRODUCTION

The securities market plays an important role in the economy of any country. The possibilities of the securities market is attracting more and more investment in this sector of the market economy. In this context analysis and forecasting of possible profits and risks incurred by the investor in the management of their portfolio becomes relevant [1]. The essence of portfolio investment involves distributing investment potential between different groups of assets, as it is impossible to find a security that is both highly reliable and highly profitable. Depending on the purpose and tasks that are initially in the formation of a portfolio, certain percentage ratio between the different types of assets, which make portfolio of the investor, is chosen [2].

After analyzing the stock market, the investor can choose the asset and invest there funds, but investing all his capital in only one security, the investor dooms itself on either a known low yields, or a known high risk.

Consequence of the second conclusion is the need for diversification of capital between different assets. The distribution of funds under the various securities leads to the formation of the portfolio securities and thus an investor can achieve an acceptable level of risk and return of investments. This is the main advantage of portfolio investments compared with investments in individual securities [3].

The goal of building an investment portfolio is to determine the optimal portfolio structure of the alternatives if there is evidence of return on assets, their degree of riskiness expressed by the standard deviation or variance of the distribution of returns, the correlation of assets in relation to each other; investor preferences. Innovations in the capital market of financial instruments and financial products, methods of calculation and distribution of investment income on retirement savings and other problems of pension fund activity in the capital market are discussed in the writings of U.F. Sharpe, A. Marshall, P. Diamond, Kh. Pinera, I. Gerar, N. Malyutina [4].

At its essence, the CAPM is a mathematical model used to determine the prices of financial assets (securities), taking into account their associated risk and the level of income. The model is based on the assumption that investors demand a higher return for increased risk (risk premium) [5].

### MATERIALS AND METHODS

The purpose of asset management of SPF is their growth by investing those assets in securities and other financial instruments. The management company takes on the responsibility for the allocation of funds to different asset classes (stocks, bonds and other instruments), the management of a portfolio of securities with a risk-based investment; active portfolio management: monitoring and implementation of arbitrage opportunities [6].

From the point of view of the theory of portfolio investment, it becomes possible to assess how the private management companies dispose of SPF assets, what is the proportion of the shares of different types, bonds and other types of securities as well as cash and savings and other assets they would like to have in their portfolio.

To estimate the cost of common stock using pricing models for capital assets (CAPM):

- Step 1: Assess the risk-free rate of return.
- Step 2: We evaluate the expected future market risk premium.
- Step 3: We evaluate the beta coefficient of the company shares and use it as a risk index of shares.
- Step 4: Substitute the previous values in the CAPM formula, we obtain an estimate of return of shares which we are interested in [7].

There is also another interpretation of the model, according to which, if the security stabilizes the portfolio, that is, making it more correlated with the market, then, in this case, the paper returns are similar to the market [8].

The model of estimates of long-term assets is as follows:

$$R_i = R_f + \beta(R_m - R_f)$$

- $R_i$  = Expected return on risk-free securities;
- $R_f$  = The expected return on the stock market as a whole, %;

- $\beta$  = Beta, coefficient of the company, share of a unit;
- $R_m$  = Return on risk-free securities.

Index  $(R_m - R_f)$  the premium (bid) for the market risk of capital investments in risky securities, stocks and bonds of the company.

The coefficient  $\beta$  measures the sensitivity of the security return to profitability of the market index and is a reflection of the systematic risk.

If  $\beta > 1$ , then the shares of the company are more sensitive to systematic risk than the stock market in average. Therefore, it is more risky to invest in this venture than into the average company operating in the market.

If  $\beta < 1$ , then the share price of the company is less dependent on the general market factors and consequently, the risk of investment is less than the risk inherent in the average for this market [9].

In international practice, the coefficient  $\beta$  is calculated by analyzing the statistical information of the stock market. This work is carried out by specialized firms. The data on the coefficient B are published in a number of financial sources and in some periodicals, analyzing the stock markets [10].

**The Main Part:** Currently, in the era of globalization of the economic systems, the study of the behavior of a portfolio of securities on the basis of computational experiments in order to ensure a stable and sustainable pension systems, depending on changes in the situation in world financial markets is relevant for decision-making, with the formation of the financial portfolios of risky pension fund assets.

Let  $g^0 = (g_1^0, g_2^0, \dots, g_n^0)$  – accounting equilibrium volume of assets (securities) in the structures of the financial portfolio of pension fund;

$r^0 = (r_1^0, r_2^0, \dots, r_n^0)$  – an equilibrium price (yield) of assets at current values;

$g = (g_1, g_2, \dots, g_n)$  – endogenous equilibrium volume of assets in the financial structures of the pension fund portfolio;

$r = (r_1, r_2, \dots, r_n)$  – endogenous equilibrium price of assets at current values

$Y = (Y_1, Y_2, \dots, Y_n)$  – final demand for assets in the structures of the financial portfolio of pension fund risk-based;

$w = (w_1, w_2, \dots, w_n)$  – price of demand for assets at current values;

$\sigma = \sigma_{ij}$  - covariance matrix  $i, j = \overline{1, n}$ :  $\sigma_{ij}$  - covariance between  $i$ - and  $j$ - assets;

$B = (B_1, B_2, \dots, B_n)'$  - balancing volume demand of assets in the securities market;

$R = (R_1, R_2, \dots, R_n)$  - balancing supply price of assets in the securities' market;

$G = (G_1, G_2, \dots, G_n)'$  - balancing volume of supply of assets in the securities market;

$V = (V_1, V_2, \dots, V_n)$  - balancing demand price of assets in the securities market.

Then the conditions for stability and growth in assets of financial companies and pension funds on the basis of calculated model of financial portfolios of risky assets in the CAPM (Capital Assret Pricing Model) system:

$$r^0 Y = rY + (r^0 - r)\sigma g, \tag{1}$$

are reduced to linear programming problems:

The direct problem (1): it is required to find the unknown vector - column,  $g = (g_1, g_2, \dots, g_n)'$ , minimizing the linear form,

$$(r^0 + R)\sigma g \rightarrow \min, \tag{2}$$

under the constraints:

$$\sigma g \geq Y + B, \tag{3}$$

$$g \geq 0, g_j \geq 0, j = \overline{1, n} \tag{4}$$

The dual task (1): it is required to find an unknown vector  $r = (r_1, r_2, \dots, r_n)$ , minimizing the linear form

$$(r^0 - r)(Y + B) \rightarrow \min, \tag{5}$$

under the constraints:

$$(r + R)\sigma \geq 0, \tag{6}$$

$$r^0 - r \geq 0, r_j^0 - r_j \geq 0, j = \overline{1, n} \tag{7}$$

Then the conditions for stability and growth in assets of financial companies and pension funds on the basis of

estimates of accounting computational model of financial portfolios of risky assets in the CAPM (Capital Assret Pricing Model) system:

$$wg^0 = wg + r\sigma(g^0 - g), \tag{8}$$

are reduced to linear programming tasks:

The direct task (8): it is required to find an unknown vector  $r = (r_1, r_2, \dots, r_n)$  - minimizing the linear form

$$r\sigma(g^0 + G) \rightarrow \min \tag{9}$$

under the constraints:

$$r\sigma \geq w + V, \tag{10}$$

$$r \geq 0, r_j \geq 0, j = \overline{1, n} \tag{11}$$

The dual task (8): it is required to find the anonymous column vector  $g = (g_1, g_2, \dots, g_n)'$ , which maximizes the linear form

$$(w + V)(g^0 - g) \rightarrow \min, \tag{12}$$

under the constraints:

$$\sigma(g + G) \geq 0, \tag{13}$$

$$g^0 - g \geq 0, g_j^0 - g_j \geq 0, j = \overline{1, n} \tag{14}$$

On the basis of described above discount computational model of financial portfolios of risky assets in the CAPM system (1) - (14), we make the placement of investment funds on the stock market, for example, quotes of trade of SPF securities on the secondary market SPC (Figure 1) as database of Excel, using computational experiments with package of applied programs connected to Windows [11].

It helps to make effective managerial decisions for SPF on the basis of different scenario of financial markets development. We find covariance matrix  $\sigma$ , which is calculated by Excel "Data", "Data analysis", "Covariance" (Figure 2).

Using special tool "Search of solution" we get optimal decision of direct task (2)-(4) (Figure 3).

We get optimal solution for dual task as well (5)-(7) (Figure 4).

	A	B	C	D	E	F	G	
1	Quotation of SPF securities trade on the secondary market							
2	Date	Par value of securities	Emission of securities (thousand units)	Demand for securities (thousand units)	ex change rate of securities	supply price (thousand tenge)	Demand price (thousand tenge)	
3	01.04.2012	1000	10	10	1,05	10000	10500	
4	01.04.2012	1500	2	2	1,07	3000	3210	
5	02.04.2012	500	4	3	0,98	2000	1470	
6	05.04.2012	100	6	4	0,97	600	388	
7	05.04.2012	5000	3	3	1,12	15000	16800	
8	06.04.2012	10000	2	2	1,06	20000	21200	
9	07.04.2012	2000	1	1	1,09	2000	2180	
10	08.04.2012	15000	1	1	1,12	15000	16800	
11	08.04.2012	5000	5	5	1,01	25000	25250	
12	09.04.2012	500	6	4	1,02	3000	2040	
13	12.04.2012	1000	3	2	1,02	3000	2040	
14	13.04.2012	2000	5	3	1,00	10000	6000	
15	14.04.2012	500	4	3	0,98	2000	1470	
16	15.04.2012	100	6	4	0,97	600	388	
17	15.04.2012	5000	3	3	1,10	15000	16500	
18	16.04.2012	500	2	2	1,06	1000	1060	
19	19.04.2012	500	1	1	1,07	500	535	
20	20.04.2012	2000	5	5	1,01	10000	10100	
21	21.04.2012	500	6	4	1,02	3000	2040	
22	22.04.2012	1000	3	2	1,02	3000	2040	
23	23.04.2012	500	4	3	0,98	2000	1470	
24	26.04.2012	500	1	1	1,15	500	575	
25	27.04.2012	1000	5	5	1,01	5000	5050	
26	28.04.2012	2000	5	3	1,00	10000	6000	
27	29.04.2012	1200	6	4	0,97	7200	4656	
28	30.04.2012	5000	3	3	1,12	15000	16800	
29	Total:	Price of supply (thousand tenge)					183400	
30	Total:	Price of demand (thousand tenge)						176562

Fig. 1: Quotation of SPF securities trade on the secondary market  
Source: Kazakhstani stock exchange

	A	B	C	D	E	F	G	H	I	J
1	Covariance matrix									
2		100	500	1000	1200	1500	2000	5000	10000	15000
3	100	3,11688	-0,7273	-0,5455	-0,1558	-0,0519	-0,4156	1,3507	-0,0519	-0,026
4	500	-0,7273	4,30303	-1,2727	-0,3636	-0,1212	-0,9697	-0,8485	-0,1212	-0,0606
5	1000	-0,5455	-1,2727	5,855	-0,2727	0,86147	-0,7273	-0,6364	-0,0909	-0,0455
6	1200	-0,1558	-0,3636	-0,2727	1,6364	-0,026	-0,2078	-0,1818	-0,026	-0,013
7	1500	-0,0519	-0,1212	0,8615	-0,026	0,18182	-0,0693	-0,0606	-0,0087	-0,0043
8	2000	-0,4156	-0,9697	-0,7273	-0,2078	-0,0693	3,06494	-0,4848	-0,0693	-0,0346
9	5000	1,3507	-0,8485	-0,6364	-0,1818	-0,0606	-0,4848	2,05195	-0,0606	0,2078
10	10000	-0,0519	-0,1212	-0,0909	-0,026	-0,0087	-0,0693	-0,0606	0,18182	-0,0043
11	15000	-0,026	-0,0606	-0,0455	-0,013	-0,0043	-0,0346	0,20779	-0,0043	0,0455

Fig. 2: Covariance matrix

	A	B	C	D	E	F	G	H	I	J
12	r0+w	0,9570	1,0100	1,0170	0,9540	1,0300	1,0031	1,0530	1,0350	1,1040
13	y	g	sg	Y	B	Y+B	115,332	Target function		
14	8	12	16,6754	6,8745	9,8009	16,6754	-1667,54			
15	21	28	54,9091	9,0455	45,8636	54,9091	-27454,55			
16	19	21	60,0865	71,6125	-11,5260	60,0865	-60086,50			
17	4	6	-13,9480	-7,6104	-6,3377	-13,9481	16737,60			
18	2	2	12,3030	13,1472	-0,8442	12,3030	-18454,50			
19	12	16	-6,7185	-6,8311	0,1126	-6,7185	13437,00			
20	14	14	-1,0692	6,7446	-7,8139	-1,0693	5346,00			
21	2	2	-7,6970	-4,9481	-2,7489	-7,6970	76970,00			
22	1	1	-0,6580	0,7164	-1,3745	-0,6581	9870,00			
23	r0	0,97	1,0211	1,0332	0,970	1,07	1,0117	1,0764	1,06	1,12
24	r0-r	0,013	0,0111	0,0162	0,016	0,04	0,0086	0,0234	0,025	0,016

Fig. 3: Results for direct task

	A	B	C	D	E	F	G	H	I	J	K
1	r	0,957	1,010	1,017	0,954	1,030	1,003	1,053	1,035	1,104	16,6754
2	r0	0,9700	1,0211	1,0332	0,9700	1,0700	1,0117	1,0764	1,0600	1,1200	54,9091
3	rs	2,4143	-0,1750	3,2303	0,2994	0,7194	0,0677	1,3936	-0,2480	0,0806	60,0865
4	(r0+w)s	2,4143	-0,1750	3,2303	0,2994	0,7194	0,0677	1,3936	-0,2480	0,0806	-13,9480
5		115,3329	Target function								12,3030
6	r0+w	0,9570	1,0100	1,0170	0,9540	1,0300	1,0031	1,0530	1,0350	1,1040	-6,7185
7											-1,0692
8											-7,6970
9											-0,6580

Fig. 4: Results for dual task

**CONCLUSION**

Determine the profitability of the financial instruments in the Kazakhstani market is difficult, because no model can take into account all the factors affecting prices. But with the help of the CAPM the degree of profitability dependence of the assets on the market returns can be determined. If the model includes adequate assessment of the market and the risk-free return, so the predicted return of the asset may be an indicator for investors in deciding to buy or sell certain assets.

Output. Thus, the placement of SPF assets in financial portfolios by structure  $g = (12;28;21;6;2;16;14;2;1)$  and by the discount price  $r = (0,97;1,0211;1,0332;0,97;1,07;1,0117;1,0764;1,06;1,12)$  in the CAPM (Figure 5), provides a gain of \$ 552.6214 thousand tenge Target function

On the final stage, when evaluating effectiveness of portfolio investment, preferences are focusing on the level of achieving individual goals of the investor, consequently it is necessary to differentiate criteria & indicators of effectiveness depending on chosen approach to portfolio formation. Investment portfolio which fully meets the goal, according to its type as well as composition of financial instruments, is effective portfolio.

	A	B	C	D
1	CAPM model (in structures)			
2	$r0(Y+B) =$	$r(Y+B)$	"+"	$(r0-r)sg$
3	117,1151	115,3322		1,7829
4				
5	CAPM model (in nominal)			
6	$r0(Y+B) =$	$r(Y+B)$	"+"	$(r0-r)sg$
7	16747,678	16197,417		552,6214

Fig. 5: Model CAPM

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