UDC 550.3
IRSTI 37.31

THERMAL DEFORMATION, COMPLEX BENDING AND STRETCHING OF LITHOSPHERIC PLATE IN TECTONIC FORCE FIELD

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Plate tectonics implies the rigidity of near-surface Earth rocks and their elastic behavior at geological scales of time. The plates are subjected to tectonic and thermogradient forces resulting in their bending, stretching and thermal deformation. The article considers the new setting of the problem of complex bending and stretching of the axisymmetric lithospheric plate of variable thickness caused by both force effects and uneven heating. The mathematically considered problem is reduced to a differential equation with variable coefficients, the analytical solution of which was first possible to obtain by the method of partial sampling.

Key words: lithospheric plate, complex bend, stretching, transverse forces, temperature field.

ТЕКТОНИКАЛЫҚ КҮШТЕР ӨРІСІНДЕГІ ЛИТОСФЕРАЛЫҚ ПЛИТАНЫҢ ЖЫЛУ ДЕФОРМАЦИЯСЫ, КҮРДЕЛІ ИІЛУ ЖӘНЕ СОЗЫЛУЫ

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Геологиялық уақыт масштабында Жер бетіне жақын жыныстарының қаттылығын және олардың серпімді қасиетін плиталар тектоника білдіреді. Тектоникалық және термоградиент күштердің әсерінен плиталардың иілу, созылуға және жылу деформациясы қабілеттігі артады. Мақалада күш әсерінен, сондай-ақ біркелкі емес қыздырудан күрделі иілу және созылу қалыңдығы айнымалы осьсимметриялық литосфера плитасына жаңа міндет-тер қою қарастырылған. Қарастырылып отырған есеп математикалық түрде айнымалы коэффициенттік дифференциалдық теңдеуге әкеледі, оның аналитикалық шешімі алғашқы рет ішінара дискреттеу әдісімен табылды.

Негізгі сөздер: литосфералық плита, күрделі иілу, созылу, көлденең күш, температуралық өріс.

ТЕПЛОВАЯ ДЕФОРМАЦИЯ, СЛОЖНЫЙ ИЗГИБ И РАСТЯЖЕНИЕ ЛИТОСФЕРНОЙ ПЛИТЫ В ПОЛЕ ТЕКТОНИЧЕСКИХ СИЛ

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Тектоника плит подразумевает жесткость приповерхностных пород Земли и их упругое поведение в геологических масштабах времени. Плиты испытывают воздействие тектонических и термоградиентных сил, приводящее к их изгибу, растяжению и тепловой деформации. В статье рассмотрена новая постановка задачи о сложном изгибе и растяжении осесимметричной литосферной плиты переменной толщины, обусловленном как силовыми воздействиями, так и неравномерным нагревом. Математически рассматриваемая задача

сводится к дифференциальному уравнению с переменными коэффициентами, аналитическое решение которой впервые удалось получить методом частичной дискретизации.

Ключевые слова: литосферная плита, сложный изгиб, растяжение, поперечные силы, температурное поле.

Introduction

The plate tectonics model implies the stiffness of near-surface rocks and their elastic behavior at geological scales of time. The plates are subjected to tectonic and thermogradient forces resulting in their bending, stretching and thermal deformation. Thin resilient surface plates form a lithosphere which floats on the underlying relative liquid mantle. The plates experience various loads, such as the weight of volcanoes and seamounts, so that they bend. For the study of such geological phenomena, the theory of bending of plates under the influence of applied forces and moments of forces is applicable. With this theory it is also possible to explain the occurrence of series of folds in mountain belts, as the formation of folds can be seen as deformations of elastic plates under the action of horizontal compressive forces. The theory of plate bending is used to simulate formation blasting over magmatic intrusions [1-3].

Objects and research methods

The problem of joint bending and stretching of non-uniform non-composite plate of variable thickness in non-uniform temperature field is considered. Partial sampling is used to find an analytical solution to the problem [4]. The basic differential equations of quasi-static equilibrium are reduced to the next second-order differential equation with respect to angular displacement

$$\frac{d^2\vartheta}{dr^2}+\left(\frac{1}{r}+\frac{1}{D_M}\frac{dD_M}{dr}\right)\frac{d\vartheta_1}{dr}+\left(\frac{v}{rD_M}\frac{dD_M}{dr}-\frac{N_r}{D_M}-\frac{1}{r^2}\right)\vartheta_1$$
$$+\frac{1}{rD_M}\left(\int q_{1z}rdr-c\right)-\frac{1+vd}{D_M dr}\left(\chi_T D_M\right)=0$$
$$\frac{d^2N_r}{dr^2}+\left(\frac{3}{r}-\frac{1}{D_N}\frac{dD_N}{dr}\right)\frac{dN_r}{dr}-F\frac{(1-v)}{rD_N}\frac{dD_N}{dr}N_{1r}\frac{1}{r}\frac{dq_r r}{dr}+\frac{q_r}{r}$$
$$\left(1+v-\frac{r}{D_N}\frac{dD_N}{dr}\right)+\frac{1-v^2}{r}D_N\frac{d\varepsilon_T}{dr}=0 \qquad (1)$$

where $\vartheta=-\frac{\partial w}{\partial r}$ - angular movement, w – deflection, D_M – cylindrical rigidity of a bend, ν – Poisson's coefficient, r – position of midplane point before its deformation, χ_T – thermal deformation due to non-uniform heating, q_r – intensity of cross forces.

An edge task is set when a circular plate of variable thickness with a rigidly embedded inner contour $r=r_1$ is loaded along its outer contour $r=r_2$. Boundary conditions will take the form

$$M_r(r_2)=0\,,\quad \vartheta(r_1)=0\,. \qquad (2)$$

Let the plate be subjected to uneven heating. In the case of linear heat propagation $\alpha_T T$ in the thickness of the plate the thermal deformation is approximated in the form of

$$\chi_T=\frac{1}{h}\sum_{j=0}^{k}\Delta\varepsilon_j r^j,\ \Delta\varepsilon_j=const \qquad (3)$$

In addition, let the plate of variable thickness rigidly embedded on the inner office $r=r_1$ be loaded uniformly distributed along the surface by transverse forces of intensity q_0 and along the contour $r=r_2$ by transverse force Q

$$q_r=\sum_{j=0}^{k}q_j r^j,\ q_j=const \qquad (4)$$

Then the general solution of equation (1) under an arbitrary law of change in plate thickness will be

$$\vartheta=B+A\int e^{-\int\xi(r)dr}dr+\int e^{-\int\xi(r)dr}\left(\int\left[\eta(r)+\varsigma(r)+\varphi(r)\right]e^{-\int\xi(r)dr}dr\right)dr \qquad (5)$$

where

$$\eta(r) = -v\sum\left[\ln\frac{D_N(r_k)}{D_{ON}}\frac{\vartheta(r_{k-1})}{r_{k-1}}\delta(r-r_{k-1}) - \ln\frac{D_M(r_k)}{D_{ON}}\frac{\vartheta(r_k)}{r_{k+1}}\delta(r-r_k)\right] -$$

$$-\sum\left[\left(\frac{1}{r_k}\right)\vartheta(r_{k-1})\delta(r-r_{k-1}) - \left(\frac{1}{r_k}\right)\vartheta(r_k)\delta(r-r_k)\right] + \qquad (6)$$

$$+\sum\psi(r_k)\left[\frac{N_r(r_{k-1})}{D_M(r_{k-1})}\vartheta(r_{k-1})\delta(r-r_{k-1}) - \frac{N_r(r_k)}{D_M(r_k)}\vartheta(r_k)\delta(r-r_{k-1})\right]$$

$$\varsigma(r) = -\frac{1}{rD_M}\left(\int q_z r dr - C\right) \quad \varphi(r) = -\frac{1+v}{D_M}\frac{d}{dr}(\chi_T D_M); \quad \xi(r) = \frac{1}{r} + \frac{1}{D_M}\frac{dD_M}{dr}. \qquad (7)$$

Arbitrary coefficients A and B are determined by boundary conditions (2).

The cylindrical bending stiffness is

$$D_M = D_0\left[1-\left(\frac{r}{r_0}\right)^{\alpha_0}\right]^{3\beta}.$$

Using the boundary conditions (2) we get

$$\vartheta(r) = 0.2\frac{Mr_0}{D_{OM}} - 0.346\frac{q_0 r_0^{\ 3}}{D_{OM}} - 0.42\frac{Qr_0^{\ 2}}{D_{OM}}\left(0.43\frac{M}{D_{OM}} - 0.04\frac{q_0 r_0^{\ 2}}{D_{OM}} - 0.236\frac{Qr_0}{D_{OM}}\right)J_1(r) +$$

$$+\frac{q_0 r_0^3}{3D_{OM}}\left[\frac{1}{2}\ln\left(1-\frac{r}{r_0}\right) + \frac{r}{r_0\left(1-\frac{r}{r_0}\right)} - \frac{1}{16\left(1-\frac{r}{r_0}\right)}\right] + \frac{Qr_0^2}{4D_{OM}\left(1-\frac{r}{r_0}\right)^2} -$$

$$-J_1(r)\left\{v\sum\left[\ln\frac{D_M(r_k)}{D_{OM}}\frac{\vartheta(r_{k-1})}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3 H(r-r_{k-1})(r-r_{k-1}) -\right.\right.$$

$$\left. -\ln\frac{D_M(r_k)}{D_{OM}}\frac{\vartheta(r_k)}{r_0}\left(1-\frac{r_k}{r_0}\right)^3 H(r-r_{k-1})(r-r_{k-1})\right] +$$

$$+\sum\left[\left(\frac{1}{r_k}\right)\vartheta(r_{k-1})\frac{r_{k-1}}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3 H(r-r_{k-1})(r-r_{k-1}) -\right.$$

$$\left.\left. -\left(\frac{1}{r_k}\right)\vartheta(r_k)\frac{r_k}{r_0}\left(1-\frac{r_k}{r_0}\right)^3 H(r-r_k)(r-r_k)\right]\right\} -$$

$$-\sum\psi(r_k)\left[\frac{N_r(r_{k-1})}{D_M(r_{k-1})}\vartheta(r_{k-1})\frac{r_{k-1}}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3 H(r-r_{k-1})(r-r_{k-1}) -\right.$$

$$\left.\left. -\frac{N_r(r_k)}{D_M(r_k)}\vartheta(r_k)\frac{r_k}{r_0}\left(1-\frac{r_k}{r_0}\right)^3 H(r-r_k)(r-r_k)\right]\right\},$$

$$\frac{d\vartheta}{dr}=\frac{0.43M-0.04q_0r_0{}^2-0.236Qr_0{}^2}{D_{OM}\,\frac{r}{r_0}\left(1-\frac{r}{r_0}\right)^3}+\frac{0.5Qr_0{}^2-0.125q_0r_0{}^2-0.166q_0r_0{}^2}{D_O\left(1-\frac{r}{r_0}\right)^3}-\frac{1}{\frac{r}{r_0}\left(1-\frac{r}{r_0}\right)^3}$$

$$\left\{v\sum\left[\ln\frac{D_M(r_k)}{D_{OM}}\frac{\vartheta(r_{k-1})}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3H(r-r_{k-1})-\ln\frac{D_M(r_k)}{D_{OM}}\frac{\vartheta(r_k)}{r_0}\left(1-\frac{r_k}{r_0}\right)^3H(r-r_{k-1})\right]+\right.$$

$$+\sum\left[\left(\frac{1}{r_k}\right)\vartheta(r_{k-1})\frac{r_{k-1}}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3H(r-r_{k-1})-\left(\frac{1}{r_k}\right)\vartheta(r_k)\frac{r_k}{r_0}\left(1-\frac{r_k}{r_0}\right)^3H(r-r_k)\right]-$$

$$\left.-\sum\psi(r_k)\left[\frac{N_r(r_{k-1})}{D_M(r_{k-1})}\vartheta(r_{k-1})\frac{r_{k-1}}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3H(r-r_{k-1})-\frac{N_r(r_k)}{D_M(r_k)}\vartheta(r_k)\frac{r_k}{r_0}\left(1-\frac{r_k}{r_0}\right)^3H(r-r_k)\right]\right\}.$$

Bending moments will have expressions

$$M_r=D_{OM}\left(1-\frac{r}{r_0}\right)^3\left\{\frac{0.43M-0.04q_0r_0{}^2-0.236Qr_0{}^2}{D_{OM}\,\frac{r}{r_0}\left(1-\frac{r}{r_0}\right)^3}+\frac{0.5Qr_0{}^2-0.125q_0r_0{}^2-0.166q_0r_0{}^2}{D_{OM}\left(1-\frac{r}{r_0}\right)^3}+\right.$$

$$+\frac{v}{r}\left(0.2\frac{Mr_0}{D_{OM}}-0.346\frac{q_0r_0{}^3}{D_{OM}}-0.42\frac{Qr_0{}^2}{D_{OM}}\left[0.43\frac{M}{D_{OM}}-0.04\frac{q_0r_0{}^2}{D_{OM}}-0.236\frac{Qr_0}{D_{OM}}\right]J_1(r)+\right.$$

$$\left.\left.+\frac{Qr_0^2}{4D_{OM}\left(1-\frac{r}{r_0}\right)}+\frac{q_0r_0^3}{3D_{OM}}\left[\frac{1}{2}\ln\left(1-\frac{r}{r_0}\right)+\frac{r}{r_0\left(1-\frac{r}{r_0}\right)}-\frac{1}{16\left(1-\frac{r}{r_0}\right)^2}\right]\right)\right)-$$

$$\frac{1}{\frac{r}{r_0}\left(1-\frac{r}{r_0}\right)^3}\cdot\left\{v\sum\left[\ln\frac{D_M(r_k)}{D_{OM}}\frac{\vartheta(r_{k-1})}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3H(r-r_{k-1})-\ln\frac{D_M(r_k)}{D_{OM}}\frac{\vartheta(r_k)}{r_0}\left(1-\frac{r_k}{r_0}\right)^3H(r-r_{k-1})\right]+\right.$$

$$+\sum\left[\left(\frac{1}{r_k}\right)\vartheta(r_{k-1})\frac{r_{k-1}}{r_0}\left(1-\frac{r_{k*1}}{r_0}\right)^3H(r-r_{k-1})-\left(\frac{1}{r_k}\right)\vartheta(r_k)\frac{r_k}{r_0}\left(1-\frac{r_k}{r_0}\right)^3H(r-r_k)\right]-$$

$$-\sum\psi(r_k)\left[\frac{N_r(r_{k-1})}{D_M(r_{k-1})}\vartheta(r_{k-1})\frac{r_{k-1}}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3H(r-r_{k-1})-\right.$$

$$\left.\left.-\frac{N_r(r_k)}{D_M(r_k)}\vartheta(r_k)\frac{r_k}{r_0}\left(1-\frac{r_k}{r_0}\right)^3H(r-r_k)(r-r_{k-1})\right]\right\}-$$

$$-\frac{v}{r}J_1(r)\left\{v\sum\left[\ln\frac{D_M(r_k)}{D_{OM}}\frac{\vartheta(r_{k-1})}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3H(r-r_{k-1})(r-r_{k-1})-\right.\right.$$

$$-\ln\frac{D_M(r_k)}{D_{OM}}\frac{\vartheta(r_k)}{r_0}\left(1-\frac{r_k}{r_0}\right)^3 H(r-r_{k-1})(r-r_{k-1})\Bigg]+$$

$$+\sum\left[\left(\frac{1}{r_k}\right)\vartheta(r_{k-1})\frac{r_{k-1}}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3 H(r-r_k)(r-r_{k-1})-\left(\frac{1}{r_k}\right)\vartheta(r_k)\frac{r_k}{r_0}\left(1-\frac{r_k}{r_0}\right)^3 H(r-r_k)(r-r_{k-1})\right]-$$

$$\sum\psi(r_k)\left[\frac{N_r(r_{k-1})}{D_M(r_{k-1})}\vartheta(r_{k-1})\frac{r_{k-1}}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3 H(r-r_k)(r-r_{k-1})-\right.$$

$$\left.\frac{N_r(r_k)}{D_M(r_k)}\vartheta(r_k)\frac{r_k}{r_0}\left(1-\frac{r_k}{r_0}\right)^3 H(r-r_k)\right]\Bigg\},$$

$$M_\theta=D_{OM}\left(1-\frac{r}{r_0}\right)^3\left\{\frac{0.43M-0.04q_0r_0^{\,2}-0.236Qr_0^{\,2}}{D_{OM}\frac{r}{r_0}\left(1-\frac{r}{r_0}\right)^3}+\frac{0.5Qr_0^{\,2}-0.125q_0r_0^{\,2}-0.166q_0r_0^{\,2}}{D_{OM}\left(1-\frac{r}{r_0}\right)^3}+\right.$$

$$+\frac{1}{r}\left(0.2\frac{Mr_0}{D_{OM}}-0.346\frac{q_0r_0^{\,3}}{D_{OM}}-0.42\frac{Qr_0^{\,2}}{D_{OM}}+\left[0.43\frac{M}{D_{OM}}-0.04\frac{q_0r_0^{\,2}}{D_{OM}}-0.236\frac{Qr_0}{D_{OM}}\right]J_1(r)+\right.$$

$$\left.+\frac{Qr_0^2}{4D_{OM}\left(1-\frac{r}{r_0}\right)}+\frac{q_0r_0^3}{3D_{OM}}\left[\frac{1}{2}\ln\left(1-\frac{r}{r_0}\right)+\frac{r}{r_0\left(1-\frac{r}{r_0}\right)}-\frac{1}{16\left(1-\frac{r}{r_0}\right)^2}\right]\right)-$$

$$\frac{v}{\frac{r}{r_0}\left(1-\frac{r}{r_0}\right)^3}\cdot\left\{v\sum\left[\ln\frac{D_M(r_k)}{D_{OM}}\frac{\vartheta(r_{k-1})}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3 H(r-r_{k-1})-\ln\frac{D_M(r_k)}{D_{OM}}\frac{\vartheta(r_k)}{r_0}\left(1-\frac{r_k}{r_0}\right)^3 H(r-r_k)\right]+\right.$$

$$+\sum\left[\left(\frac{1}{r_k}\right)\vartheta(r_{k-1})\frac{r_{k-1}}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3 H(r-r_{k-1})-\left(\frac{1}{r_k}\right)\vartheta(r_k)\frac{r_k}{r_0}\left(1-\frac{r_k}{r_0}\right)^3 H(r-r_k)\right]-$$

$$\left.-\sum\psi(r_k)\left[\frac{N_r(r_{k-1})}{D_M(r_{k-1})}\vartheta(r_{k-1})\frac{r_{k-1}}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3 H(r-r_{k-1})-\frac{N_r(r_k)}{D_M(r_k)}\vartheta(r_k)\frac{r_k}{r_0}\left(1-\frac{r_k}{r_0}\right)^3 H(r-r_k)\right]\right\}-$$

$$-\frac{1}{r}J_1(r)\left\{v\sum\left[\ln\frac{D_M(r_k)}{D_{OM}}\vartheta(r_{k-1})\frac{1}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3 H(r-r_{k-1})(r-r_{k-1})-\right.\right.$$

$$-\ln \frac{D_M(r_k)}{D_{OM}} \vartheta(r_k)\frac{1}{r_0}\left(1-\frac{r_k}{r_0}\right)^3 H(r-r_{k-1})(r-r_{k-1})\Bigg]+$$

$$+\sum\left[\left(\frac{1}{r_k}\right)\vartheta(r_{k-1})\frac{r_{k-1}}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3 H(r-r_{k-1})(r-r_{k-1})-\left(\frac{1}{r_k}\right)\vartheta(r_k)\frac{r_k}{r_0}\left(1-\frac{r_k}{r_0}\right)^3 H(r-r_k)(r-r_{k-1})\right]-$$

$$\sum\psi(r_k)\Bigg[\frac{N_r(r_{k-1})}{D_M(r_{k-1})}\vartheta(r_{k-1})\frac{r_{k-1}}{r_0}\left(1-\frac{r_{k-1}}{r_0}\right)^3 H(r-r_{k-1})(r-r_{k-1})-$$

$$\frac{N_r(r_k)}{D_M(r_k)}\vartheta(r_k)\frac{r_k}{r_0}\left(1-\frac{r_k}{r_0}\right)^3 H(r-r_k)\Bigg]\Bigg\}.$$

Results and their discussion

The application of the partial sampling method has made it possible to solve the problem for any law of changing mechanical characteristics. On the basis of the found solution and numerical analysis for stress-deformed state of the lithospheric plate under the action of the normal stretching effort and transverse forces symmetrically distributed in the middle plane, as well as a result of temperature heating, patterns of change of radial bending moments and circumferential bending moments are found, which are shown in the form of graphs in Figures 1.

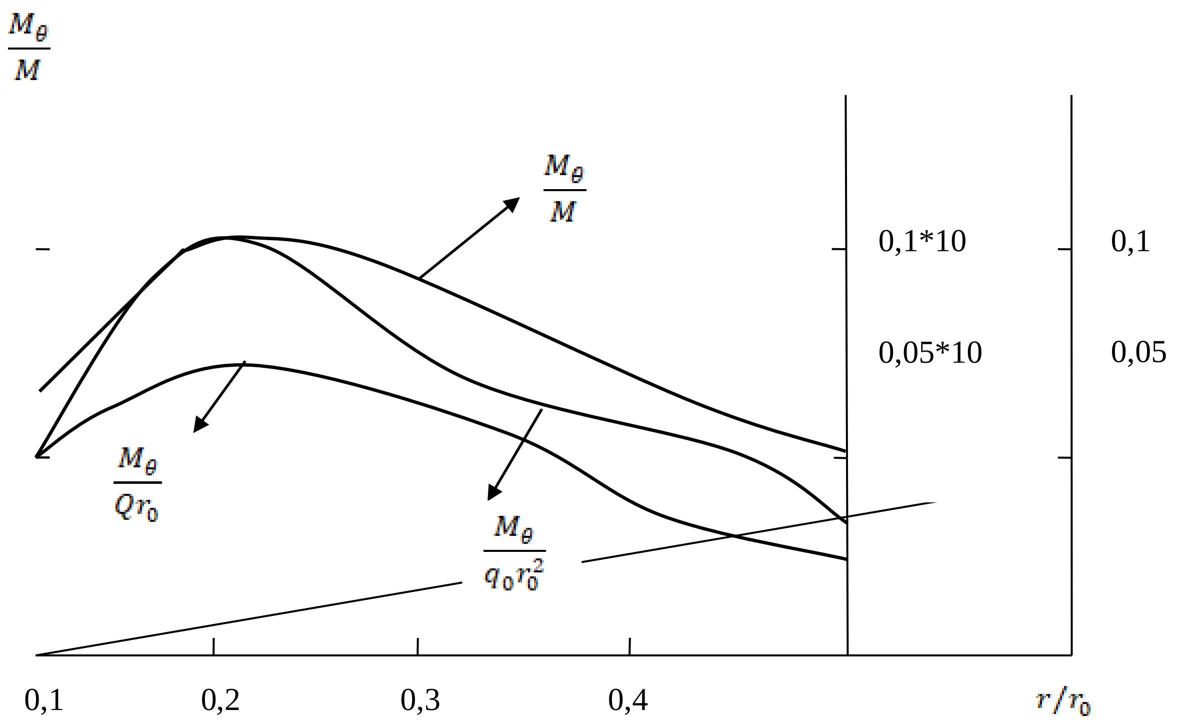

Figure 1 - Circumferential bending moments M_θ at joint bending and stretching of non-uniform plate of variable thickness.

Conclusion

New model of stress-strain state of axisymmetric lithospheric plate of exponential profile in non-uniform temperature field and under action of normal stretching effort and transverse forces symmetrically distributed in the middle plane is proposed.

To solve the nonlinear differential equation with non-uniform bending coefficients of the lithospheric plate, the partial sampling method was applied for the first time.

Obtained regularities of change of radial bending moments and circumferential bending moments under action of normal stretching effort and transverse forces symmetrically distributed in the middle plane, and also as a result of temperature heating characterize stressed-deformed state of plate. Graphical analysis indicates the nonlinear nature of their distribution.

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